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Robust stability of system with one input and one output in a class of catastrophe "hyperbolic umbilic"

Beisenbi Mamyrbek[#1],
Department of System analysis and Control
L.N. Gumilev Eurasian National University
Republic of Kazakhstan, Astana city 11, Pushkin str.
[1]beisenbi@mail.ru

Mukataev Nurlan[#2]
Department of System analysis and Control
L.N. Gumilev Eurasian National University
Republic of Kazakhstan, Astana city 11, Pushkin str.
[2]sgpi@mail.ru.

***Abstract* - In the article there is a new approach to create control systems for objects with uncertain parameters in the form of three-parametrical structurally stable mappings from the theory of the catastrophe, allowing to synthesize the highly effective control systems, possessing extremely wide area of robust stability is offered.**

Research of robust stability control systems is based on a new approach to post-rhenium of the A.M Lyapunov's functions.

The method of creation of a control system with an increased potential of robust stability is stated.

***Keywords* - control system, uncertainty of parameters, robust stability, structurally stable mappings, stability area.**

I. INTRODUCTION

The research problem of stability occupies one of the central places at creation of control systems by technical objects and technological processes which are widely applied practically in all spheres of production and equipment: in mechanical engineering, an energy drink, electronic, chemical, biological, metallurgical and textile industry, transport, robotics, aircraft, space systems, high-precision military technology and equipment, etc.

Now it is conventional that the majority of real control systems make the functions in the conditions of this or that degree of uncertainty. Thus uncertainty can be caused by ignorance of true values of parameters of objects of control and their unpredictable change in time (in operating process). Therefore the problem of robust stability is one of the most actual in the theory of control and represents great practical interest. In the general statement it occupies the post in the indication of restrictions on changes of parameters of system at which stability remains. It is clear, that these restrictions are determined by stability area by uncertain parameters of object and established parameters of a control unit.

Known methods of [1-4] creation of control systems with uncertain parameters are generally devoted to determination of robust stability of system with the set structure with linear laws of control or radiant nonlinear (relay) characteristics and don't allow to project a control system with rather wide area of robust stability in the conditions of big uncertainty of parameters of object of control and drift of their characteristic in big limits.

In [5] the special attention is paid for dynamic systems in which development of processes of self-organization in physical and chemical and biological systems are considered.

Models of these systems are represented in the form of structurally stable mappings from the theory of catastrophe [6,7] and are being investigated as universal mathematical model of development and self-organization in wildlife. In this regard the to construct system of automatic control in the conditions of big uncertainty in a class of structurally stable mappings with the mathematical models accompanying difficult behavior of system, namely having a set of consistently steady decisions represent a certain interest.

This research is devoted to actual problems of creation of a robust steady control system by linear dynamic objects with uncertain parameters with approach to a choice of laws of control in a class of the three-parametrical structurally stable mappings, allowing to maximize the potential of robust stability and indicators of quality of a control system [8-14].

The concept of creation of a control system with an increased potential of robust stability by dynamic objects is based on results of the theory of catastrophe [6,7] where as the main result the major structurally stable mappings are received. They are limited and directly defined by number of operating parameters.

Universal research method of stability of dynamic systems is the functions method of A.M.Lyapunov [15,16]. As the instrument of research in Lyapunov's method some special continuously differentiable are being used, turning at the beginning of coordinates in zero the functions called by function of Lyapunov. Application of this method restrains by lack of universal approach to create the function of Lyapunov. It is necessary to remind that the mistake in a choice and failure in construction the necessary function of Lyapunov doesn't mean instability of system: it points only to failure at creation of Lyapunov's functions [15-17].

Now there are no universal scientific provisions on development and research of robust stability of nonlinear control systems with rather wide area of stability, providing the best protection against development in mode system "determined chaos" regime. Therefore it is obviously important in conditions of big uncertainty in parameters of objects of control with chaos generation, to construct system of automatic control in a class of structurally stable mappings [6,7] with the mathematical models accompanying difficult behavior of system [8-14], namely having consistently steady decisions.

Research of the last years showed that methods of creation of the A.M. Lyapunov's functions can be applied to studying of a robustness of linear or nonlinear control systems with success, based on geometrical interpretation of A.M. Lyapunov's theorems in space of states [18-21].

In article problems of creation of control systems with an increased potential of robust stability with new approach to research of a robustness on a method of A.M.Lyapunov's functions [18-21] are considered.

II. Mathematical formulation of a model

Let's consider the stationary closed control system with one output and one input, described by the state equation,

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad y = x_1(t) \tag{1}$$

where $x(t) \in R^n$ - the vector of a condition of object; $u(t) \in R^1$ - scalar function of operating influences; $A \in R^{nxn}$ – matrix of object of control with uncertain parameters of dimension $n \times n$, $B \in R^{mx1}$ – matrix of control of dimension $m \times 1$. Matrixes A and B have the following appearance:

$$A = \begin{Vmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{Vmatrix}, \quad B = \begin{Vmatrix} 0 \\ 0 \\ \vdots \\ b_n \end{Vmatrix}$$

The management law in the closed contour is set in the form of the sum of three-parametrical structurally stable mappings (catastrophe "hyperbolic umbilic"):

$$\begin{aligned} u(x) = & -x_2^3 + 3x_2x_1^2 = k_{12}(x_1^2 + x_2^2) + k_2x_2 + k_1x_1 - x_4^3 + \\ & + 3x_4x_3^2 - k_{34}(x_4^2 + x_3^2) + k_4x_4 + k_3x_3 -, \dots, -x_n^3 + \\ & + 3x_nx_{n-1}^2 - k_{n-1,n}(x_n^2 + x_{n-1}^2) + k_nx_n + k_{n-1}x_{n-1} \end{aligned} \tag{2}$$

The system (1) in expanded form registers:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \ \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n\left[3x_2x_1^2 - x_2^3 - k_{12}(x_1^2 + x_2^2) + (k_1 - a_n)x_1 + (k_2 - a_{n-1})x_2 + \right. \\ + 3x_4x_3^2 - x_4^3 - k_{34}(x_3^2 + x_4^2) + (k_3 - a_{n-2})x_3 + (k_4 - a_{n-3})x_4 +, \dots, + \\ + 3x_nx_{n-1}^2 - x_n^3 - -k_{n-1,n}(x_n^2 + x_{n-1}^2) + (k_{n-1} - a_2)x_{n-1} + (k_n - a_1)x_n \end{cases} \tag{3}$$

III. Stationary condition of a system

The stationary systems (installed) states are defined by the solution of the equation:

$$\begin{cases} x_{2S} = 0, \ x_{3S} = 0, \dots, x_{n-1,S} = 0, \ x_{nS} = 0, \\ 3x_{2S}x_{1S}^3 - x_{2S}^3 - k_{12}(x_{1S} + x_{2S}^2) + (k_1 - a_n)x_{1S} + (k_2 - a_{n-1})x_{2S} + \\ + 3x_{4S}x_{3S}^3 - x_{4S}^3 - k_{34}(x_{3S}^2 + x_{4S}^2) + (k_3 - a_{n-2})x_{3S} + \\ + (k_4 - a_{n-3})x_{4S} +, \dots, \ 3x_{nS}x_{n-1,S}^2 - x_{nS}^3 - k_{n-1,n}(x_{nS}^2 + x_{n-1,S}^2) + \\ + (k_{n-1} - a_2)x_{n-1,S} + (k_n - a_1)x_{nS} = 0 \end{cases} \tag{4}$$

From the equation (4) it is possible to get the stationary condition determined by the trivial decision of a system (4):

$$x_{1S} = 0, \ x_{2S} = 0, \dots, \ x_{n-1,S} = 0, \ x_{nS} = 0 \tag{5}$$

Some other stationary states will be defined by the solution of the equations

$$-k_{i,i+1}x_{iS} + k_i - a_{n-i+1} = 0, \quad x_{jS} = 0 \ when \ i \neq j, \ i = 1, \dots, n \tag{6}$$

or

$$-x_{i+1,S}^2 - k_{i,i+1}x_{i+1,} + (k_{i+1} - a_{n-i+2}) = 0, \ x_{jS} = 0, \quad when \ i+1 \neq j, \ i = 1, \dots, n \tag{7}$$

The equation (6) has the following decisions:

$$x_{iS} = \frac{k_i - a_{n-i+1}}{k_{i,i+1}}, \ x_{jS} = 0 \ where \ i \neq j, \ i = 1, \dots, n \tag{8}$$

The equations (7) at the negative $k_{i,i+1}^2 + 4(k_i - a_{n-i+2}), \ i = 1, \dots, n$ $\quad \frac{(k_{i,i+1}^2 + 4(k_i - a_{n-i+2})}{2} < 0, \ i = 1, \dots, n$ have imaginary decisions that can't correspond to any physically possible situation. When $(k_{i,i+1}^2 + 4(k_i - a_{n-i+2}) > 0,$ the equation (7) allows the following decisions:

$$x_{i+1,S}^1 = \frac{-k_{i,i+1} - \sqrt{k_{i,i+1}^2 + 4(k_i - a_{n-i+2})}}{2}, \quad x_{jS} = 0, \ for \ i+1 \neq j, \ i = 1, \dots, n \tag{9}$$

$$x_{i+1,S}^2 = \frac{-k_{i,i+1} - \sqrt{k_{i,i+1}^2 + 4(k_i - a_{n-i+2})}}{2}, \quad x_{jS} = 0, \ for \ i+1 \neq j, \ i = 1, \dots, n \tag{10}$$

IV. Research of stationary states stability

(3) we will investigate of stationary conditions stability (5), (8), (9) and (10) systems on the basis of the offered approach to methods of Lyapunov's function.

We will consider a steady state stability (5). For this purpose we will designate components of a vector of an anti-gradient of Lyapunov's vector-function

$$\frac{\partial V_1(x)}{\partial x_1}=0,\ \frac{\partial V_1(x)}{\partial x_2}=-x_2,\ \frac{\partial V_1(x)}{\partial x_3}=0,\ \dots,\ \frac{\partial V_1(x)}{\partial x_n}=0$$

$$\frac{\partial V_2(x)}{\partial x_1}=0,\ \frac{\partial V_2(x)}{\partial x_2}=0,\ \frac{\partial V_2(x)}{\partial x_3}=-x_3,\ \dots,\ \frac{\partial V_2(x)}{\partial x_n}=0$$

$$\dots\quad\dots\quad\dots\quad\dots\quad\dots$$

$$\frac{\partial V_{n-1}(x)}{\partial x_1}=0,\ \frac{\partial V_{n-1}(x)}{\partial x_2}=-x_2,\ \frac{\partial V_{n-1}(x)}{\partial x_3}=0,\ \dots,\ \frac{\partial V_{n-1}(x)}{\partial x_n}=-x_n$$

$$\frac{\partial V_n(x)}{\partial x_1}=b_nk_{12}x_1^2-b_n(k_1-a_n)x_1-3b_nx_2x_1^2$$

$$\frac{\partial V_n(x)}{\partial x_2}=b_nx_2^3+b_nk_{12}x_2^2-b_n(k_2-a_{n-1})x_2$$

$$\frac{\partial V_n(x)}{\partial x_3}=b_nk_{34}x_3^2-3b_nx_4x_3^2-b_n(k_3-a_{n-2})x_3$$

$$\frac{\partial V_n(x)}{\partial x_4}=b_nk_{34}x_4^2+b_nx_4^3-b_n(k_1-a_{n-3})x_4$$

$$\dots\quad\dots\quad\dots\quad\dots$$

$$\frac{\partial V_n(x)}{\partial x_{n-1}}=b_nk_{n-1}x_{n-1}^2-3b_nx_nx_{n-1}^2-b_n(k_{n-1}-a_2)x_{n-1}$$

$$\frac{\partial V_n(x)}{\partial x_1}=b_nx_n^3+b_nk_{n-1,n}x_n^2-b_n(k_n-a_1)x_n$$

The full derivative on time from Lyapunov's vector functions will be equal:

$$\begin{aligned}\frac{dV}{dt}&=\frac{\partial V}{\partial x}\cdot\frac{dx}{dt}=-x_2^2-x_3^2-,\dots,-x_n^2-\\&-b_n^2\left[k_{12}x_1^2-3x_2x_1^2-(k_1-a_n)x_1\right]-\\&-b_n^2\left[x_2^3+k_{12}x_2^2-(k_2-a_{n-1})x_2\right]^2-\\&-b_n^2\left[k_{34}x_3^2-3x_4x_3^2-(k_3-a_{n-2})x_3\right]^2-\\&-b_n^2\left[k_{34}x_4^2+x_4^3-(k_4-a_{n-3})x_4\right]^2-,\ \dots,-\\&-b_n^2\left[k_{n-1,n}x_{n-1}^2-3x_nx_{n-1}^2-(k_{n-1}-a_2)x_{n-1}\right]^2-\\&-b_n^2\left[x_n^3+k_{n-1,n}x_n^2-(k_n-a_1)x_n\right]^2\end{aligned}\tag{11}$$

From (11) we receive that the full derivative on time from Lyapunov's vector functions will be negative function, therefore, the sufficient condition of an asymptotic system stability (3) rather than steady state (5) is accomplished.

On a gradient vector from Lyapunov's vector functions we build unknown components of Lyapunov's vector functions in a following view

$$(V_i(x)=V_{i1}(x),\ V_{i2}(x),\dots,V_{in}(x))):$$

$$V_{11}(x)=0,\ V_{12}(x)=-\frac{1}{2}x_2^2,\ V_{13}(x)=0,\ \dots,V_{1n}(x)=0$$

$$V_{21}(x)=0,\ V_{22}(x)=0,\ V_{13}(x)=-\frac{1}{2}x_3^2,\ \dots,V_{2n}(x)=0$$

$$\dots\quad\dots\quad\dots\quad\dots\quad\dots$$

$$V_{n-1,1}(x)=0,\ V_{n-1,2}(x)=0,\ V_{n-1,3}(x)=0,\ \dots,V_{n-1,n}(x)=\frac{1}{2}x_n^2$$

$$V_{n1}(x)=\frac{1}{3}b_nk_{12}x_1^3-b_nx_2x_1^3-\frac{1}{2}b_n(k_1-a_n)x_1^2,\dots,$$

$$V_{n2}(x)=\frac{1}{4}b_nx_2^4+\frac{1}{3}b_nk_{12}x_2^3-\frac{1}{2}b_n(k_2-a_{n-1})x_2^2$$

$$V_{n3}(x)=\frac{1}{3}b_nk_{34}x_3^3-b_nx_4x_3^3-\frac{1}{2}b_n(k_3-a_{n-2})x_3^2,\dots,$$

$$V_{n4}(x)=\frac{1}{3}b_nk_{34}x_4^3+\frac{1}{4}x_4^4-\frac{1}{2}b_n(k_4-a_{n-3})x_4^2$$

$$\dots\quad\dots\quad\dots\quad\dots\quad\dots$$

$$V_{n,n-1}(x)=\frac{1}{3}b_nk_{n-1,n}x_{n-1}^3-b_nx_nx_{n-3}^3-\frac{1}{2}b_n(k_{n-1}-a_2)x_{n-1}^2$$

$$V_{n,n}(x)=\frac{1}{4}b_nx_n^4+\frac{1}{3}b_nk_{n-1,n}x_n^3-\frac{1}{2}b_n(k_n-a_1)x_n^2$$

We can present Lyapunov's function in a scalar form in a following view:

$$\begin{aligned}V(x)&=\frac{1}{3}b_nk_{12}x_1^3-b_nx_2x_1^3-\frac{1}{2}b_n(k_1-a_n)x_1^2+\frac{1}{4}b_nx_2^4+\frac{1}{3}b_nx_{12}x_2^3-\\&-\frac{1}{2}b_n(k_1-a_{n-1})x_2^2+\frac{1}{3}b_nk_{34}x_3^3-b_nx_4x_3^3-\frac{1}{2}b_n(k_3-a_{n-2}+\frac{1}{b_n})x_3^2+\\&+\frac{1}{3}b_nk_{34}x_4^3+\frac{1}{4}x_4^4-\frac{1}{2}b_n(k_4-a_{n-3}+\frac{1}{b_n})x_4^2+,\ \dots\ ,+\frac{1}{3}b_nk_{n-1,n}x_{n-1}^3-\\&-b_nx_nx_{n-1}^3-\frac{1}{2}b_n(k_{n-1}-a_2+\frac{1}{b_n})x_{n-1}^2+\frac{1}{4}b_nx_n^4+\frac{1}{3}b_nk_{n-1,n}x_n^3-\\&-\frac{1}{2}b_n(k_n-a_1+\frac{1}{b_n})x_n^2,\end{aligned}\tag{12}$$

Conditions of positive or negative definiteness of functions (12) are unevident therefore we will use the Morse's lemma from the theory of catastrophe [6,7].

The considered system (3) is in a condition of stable or unstable balance, by other words, is Morse points [6,7] i.e. the system is in stationary points x_s, where a gradient from Lyapunov's functions $\nabla V=0$ and when

$$\det V_{ij}=\left|\frac{\partial^2V(x)}{\partial x_i\partial x_j}\bigg|_{x_s}\right|\neq 0$$

that in these stationary conditions of system the lemma of the Mors is fair and guarantees existence of smooth replacement of variables, such that Lyapunov's function (12) can be locally presented by a square form:

$$V=\sum_{i=1}^{n}\lambda_ix_i^2\tag{13}$$

Here λ_i - own values of Hess's matrix

$$V_{ij} = (x_S) = \left[\frac{\partial^2 V}{\partial x_i \partial x_j}\bigg|_{x_S} \right]$$

Positive definiteness of functions of Lyapunov will be defined by signs of coefficients of a square form (10), (λ_i >0, i=1, ... , n), i.e. signs of own values of a matrix of Hess (stability matrix). Therefore, it is necessary to define Hess's matrix in a point of balance (5). We calculate the Hess's matrix for Lyapunov's functions (12) in a stationary point (5).

Let's designate:

$$\frac{\partial V(x)}{\partial x_1} = b_n (k_{12} x_1^2 - 3x_2 x_1^2 - (k_1 - a_n) x_1)$$
$$\frac{\partial V(x)}{\partial x_2} = b_n (x_2^3 + k_{12} x_2^2 - (k_2 - a_{n-1} + \frac{1}{b_n}) x_2)$$
$$\frac{\partial V(x)}{\partial x_3} = b_n (k_{34} x_3^2 - 3x_4 x_3^2 - (k_3 - a_{n-2} + \frac{1}{b_n}) x_3)$$
$$\frac{\partial V(x)}{\partial x_4} = b_n (k_{34} x_4^2 + x_4^3 - (k_4 - a_{n-3} + \frac{1}{b_n}) x_4)$$
$$\dots \quad \dots \quad \dots \quad \dots \quad \dots$$
$$\frac{\partial V(x)}{\partial x_{n-1}} = b_n (k_{n-1,n} x_{n-1}^2 - 3x_n x_{n-1}^2 - (k_{n-1} - a_2 + \frac{1}{b_n}) x_{n-1})$$
$$\frac{\partial V(x)}{\partial x_n} = b_n (x_n^3 k_{n-1,n} x_n^2 - (k_n - a_1 + \frac{1}{b_n}) x_n),$$

We calculate elements of the Hess's matrix:

$$\lambda_1 = -b_n(k_1 - a_n), \ \lambda_2 = -b_n\left(k_2 - a_{n-1} + \frac{1}{b_n}\right),$$
$$\lambda_3 = -b_n\left(k_3 - a_{n-2} + \frac{1}{b_n}\right), \ \lambda_4 = -b_n\left(k_4 - a_{n-3} + \frac{1}{b_n}\right), \ \dots ,$$
$$\lambda_n = -b_n\left(k_n - a_1 + \frac{1}{b_n}\right)$$

By the Morse lemma (12) locally we can present Lyapunov's function in locality of a steady state by the type of a square form:

$$V(x) = -b_n (k_1 - a_n) x_1^2 - b_n\left(k_2 - a_{n-1} + \frac{1}{b_n}\right) x_2^2 - -b_n\left(k_3 - a_{n-2} + \frac{1}{b_n}\right) x_3^2 - b_n\left(k_4 - a_{n-3} + \frac{1}{b_n}\right) x_4^2 -, \ \dots ,- -b_n\left(k_{n-1} - a_2 + \frac{1}{b_n}\right) x_{n-1}^2 - b_n\left(k_n - a_1 + \frac{1}{b_n}\right) x_n^2, \quad (14)$$

The necessary condition of stability of a steady state (5) will be defined by system of inequalities when b_n >0:

$$\begin{cases} k_1 - a_n < 0 \\ k_2 - a_{n-1} + \frac{1}{b_n} < 0 \\ k_3 - a_{n-2} + \frac{1}{b_n} < 0 \\ k_4 - a_{n-3} + \frac{1}{b_n} < 0 \\ \dots \quad \dots \quad \dots \\ k_{n-1} - a_2 + \frac{1}{b_n} < 0 \\ k_n - a_1 + \frac{1}{b_n} < 0 \end{cases} \quad (15)$$

Let's investigate robust stability of a steady state (8) on the basis of a method of Lyapunov's functions. The equation of a state (3) is representable in deviations of rather steady state (8).

Formally described decomposition can be presented in a following view:

$$F(X_S + x) = F(X_S) + \left(\frac{\partial F}{\partial X}\right)_{X_S} \cdot x + \frac{1}{2}\left(\frac{\partial^2 F}{\partial X \partial X}\right)_{X_S} \cdot xx + + \frac{1}{6}\left(\frac{\partial^3 F}{\partial X \partial X \partial X}\right)_{X_S} \cdot xxx + \frac{1}{24}\left(\frac{\partial^4 F}{\partial X \partial X \partial X}\right)_{X_S} \cdot xxxx + \dots$$

We calculate values of derivatives in a stationary point (8):

$$\frac{\partial F_1}{\partial x_1} = 0, \ \frac{\partial F_1}{\partial x_2} = 1, \ \frac{\partial F_1}{\partial x_3} = 0, \ \dots , \ \frac{\partial F_1}{\partial x_n} = 0$$
$$\frac{\partial F_2}{\partial x_1} = 0, \ \frac{\partial F_2}{\partial x_2} = 0, \ \frac{\partial F_2}{\partial x_3} = 1, \ \dots , \ \frac{\partial F_1}{\partial x_n} = 0$$
$$\dots \quad \dots \quad \dots \quad \dots$$
$$\frac{\partial F_{n-1}}{\partial x_1} = 0, \ \frac{\partial F_{n-1}}{\partial x_2} = 0, \ \frac{\partial F_{n-1}}{\partial x_3} = 0, \ \dots , \ \frac{\partial F_{n-1}}{\partial x_n} = 1$$
$$\frac{\partial F_n}{\partial x_2} = b_n\left[3x_1^2 - 3x_2^2 - 2k_{12}x_2 + k_2 - a_{n-1}\right]_{x_S} = -b_n(k_2 - a_{n-1})$$
$$\frac{\partial F_n}{\partial x_3} = b_n\left[6x_4 x_3 - 2k_{12}x_3 + k_2 - a_{n-2}\right]_{x_S} = -b_n(k_3 - a_{n-1})$$
$$\frac{\partial F_n}{\partial x_3} = b_n\left[3x_3^2 - 3x_4^2 - 2k_{34}x_4 + k_4 - a_{n-1}\right]_{x_S} = -b_n(k_4 - a_{n-3})$$
$$\dots \quad \dots \quad \dots \quad \dots$$
$$\frac{\partial F_n}{\partial x_n} = b_n\left[3x_{n-1}^2 - 3x_n^2 - 2k_{n-1,n}x_n + k_n - a_1\right]_{x_S} = -b_n(k_n - a_{n-1})$$
$$\frac{\partial^2 F}{\partial x_1^2} = b_n\left[6x_2 - 2k_{12}\right]_{x_S} = -2x_{12},$$
$$\frac{\partial^2 F}{\partial x_1 \partial x_2} = b_n\left[6x_1\right]_{x_S} = 6b_n(k_1 - a_n), \ \frac{\partial^2 F}{\partial x_2 \partial x_1} = b_n\left[6x_1\right] = 6b_n(k_1 - a_n)$$
$$\frac{\partial^2 F}{\partial x_2^2} = b_n\left[-6x_2 - 2k_{12}\right]_{x_S}; \ \frac{\partial^3 F}{\partial x_1^3} = 0, \ \frac{\partial^3 F}{\partial x_1^2 \partial x_2} = 6b_n$$

$$\frac{\partial^3 F}{\partial x_1^2 \partial x_2} = 6b_n, \; \frac{\partial^3 F}{\partial x_1 \partial x_2^2} = 0, \; \frac{\partial^3 F}{\partial x_2 \partial x_1^2} = 6b_n, \; \frac{\partial^3 F}{\partial x_2^3} = -6b_n,$$

$\dots \quad \dots \quad \dots \quad \dots$

$$\frac{\partial F_n}{\partial x_{n-1}} = b_n \left(6x_n x_{n-1} - 2k_{n-1,n} x_{n-1} + k_{n-1} - a_2\right)_{x_S} =$$

$$\frac{\partial F_n}{\partial x_n} = b_n \left(3x_{n-1}^2 - 3x_n^2 - 2k_{n-1,n} x_n + k_n - a_1\right)_{x_S} =$$

$$\frac{\partial^2 F_n}{\partial x_{n-1}^2} = b_n \left(6x_n - 2k_{n-1,n}\right)_{x_S} =$$

$$\frac{\partial^2 F_n}{\partial x_{n-1} \partial x_n} = b_n \left(6x_n - 2k_{n-1,n}\right)_{x_S} =$$

$$\frac{\partial^2 F_n}{\partial x_n \partial x_{n-1}} = b_n \left(6x_{n-1}\right)_{x_S} =$$

$$\frac{\partial^2 F_n}{\partial x_n^2} = b_n \left(-6x_n\right)_{x_S} =$$

$$\frac{\partial^2 F_n}{\partial x_{n-1}^2 \partial x_n} = 6b_n, \; \frac{\partial^3 F_n}{\partial x_n \partial x_{n-1}^2} = 6b_n, \; \frac{\partial^3 F_n}{\partial x_n^2 \partial x_{n-1}^2} = 0$$

The equation of a state (1) in deviations of rather stationary state (8) are being registered:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots \quad \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n [3x_2 x_1^2 - k_2^3 - k_{12}\left(x_1^2 - x_2^2\right) + 12\left(k_1 - a_n\right) x_1 x_2 - (k_2 - a_{n-1}) x_2 - \\ - (k_1 - a_n) x_1 + 3x_3 x_4 - x_4^3 - k_{34}\left(x_3^2 - x_4^2\right) + 12\left(k_3 - a_n\right) x_3 x_4 + \\ + 12(k_1 - a_n) x_3 x_4 - (k_3 - a_{n-2}) x_3 - (k_4 - a_{n-3}) x_4 +, \; \dots, \; + \\ + 3x_{n-1}^2 x_n^2 - x_n^3 - k_{n-1,n}(x_{n-1}^2 + x_n^2) + 12\left(k_n - a_1\right) x_{n-1} x_n - \\ - (k_{n-1} - a_2) x_{n-1} - \left(k_n - a_1\right) x_n] \end{cases}$$

The full derivative on time from Lyapunov's vector functions will be determined by:

$$\begin{aligned} \frac{dV}{dt} = \frac{\partial V}{\partial x} \cdot \frac{dx}{dt} = \frac{1}{2} x_2^2 - \frac{1}{2} x_3^2 - \frac{1}{2} x_4^2, \; \dots \; , -\frac{1}{2} x_n^2 - \\ - b_n^2 \left[3x_2 x_1^2 - k_{12} x_1^2 + 6(k_1 - a_n) x_1 x_2 - (k_1 - a_n) x_1\right]^2 - \\ - b_n^2 \left[- x_2^3 + k_{12} x_2^2 + 6(k_1 - a_n) x_1 x_2 - (k_2 - a_{n-1}) x_2\right]^2 - \\ - b_n^2 \left[3k_4 x_3^2 - k_{34} x_2^2 + 6(k_3 - a_{n-2}) x_1 x_2 - (k_3 - a_{n-2}) x_3\right]^2 - \\ - b_n^2 \left[- x_4^2 - k_{34} x_4^2 + 6(k_3 - a_{n-2}) x_3 x_4 - (k_4 - a_{n-3}) x_4\right]^2 -, \; \dots \; ,- \\ - b_n^2 \left[3x_n x_{n-1}^2 - k_{n-1,n} x_{n-1}^2 + 6(k_n - a_1) x_n x_{n-1} - \left(k_{n-1} a_2\right) x_{n-1}\right]^2 - \\ - b_n^2 \left[- x_n^3 - k_{n-1} x_n^2 + 6(k_n - a_1) x_n x_{n-1} - (k_n - a_1) x_n\right]^2 \end{aligned} \qquad (16)$$

Scalar function (16) always will be negative, therefore, the sufficient condition of an asymptotic stability of a state (8) will be always accomplished.

Let's will designate components of a gradient vector from Lyapunov's vector functions:

$$\frac{\partial V_1(x)}{\partial x_1} = 0, \; \frac{\partial V_1(x)}{\partial x_2} = -x_2, \; \frac{\partial V_1(x)}{\partial x_3} = 0, \; \dots \; , \; \frac{\partial V_1(x)}{\partial x_n} = 0$$

$$\frac{\partial V_2(x)}{\partial x_1} = 0, \; \frac{\partial V_2(x)}{\partial x_2} = 0, \; \frac{\partial V_2(x)}{\partial x_3} = -x_3, \; \dots \; , \; \frac{\partial V_2(x)}{\partial x_n} = 0$$

$\dots \quad \dots \quad \dots \quad \dots \quad \dots$

$$\frac{\partial V_{n-1}(x)}{\partial x_1} = 0, \; \frac{\partial V_{n-1}(x)}{\partial x_2} = 0, \; \frac{\partial V_{n-1}(x)}{\partial x_3} = 0, \; \dots \; , \; \frac{\partial V_{n-1}(x)}{\partial x_n} = -x_n$$

$$\frac{\partial V_{n-1}(x)}{\partial x_1} = 0, \; \frac{\partial V_{n-1}(x)}{\partial x_2} = 0, \; \frac{\partial V_{n-1}(x)}{\partial x_3} = 0, \; \dots \; , \; \frac{\partial V_{n-1}(x)}{\partial x_n} = -x_n$$

$$\frac{\partial V_n(x)}{\partial x_1} = b_n \left(- 3x_2 x_1^2 + k_{12} x_1^2 - 6(k_2 - a_n) x_1 x_2 + \left(k_1 - a_n\right) x_1\right)$$

$$\frac{\partial V_n(x)}{\partial x_2} = b_n \left(x_2^3 + k_{12} x_2^2 - 6(k_1 - a_n) x_1 x_2 + \left(k_2 - a_{n-1}\right) x_2\right)$$

$\dots \quad \dots \quad \dots \quad \dots \quad \dots$

$$\frac{\partial V_n(x)}{\partial x_{n-1}} = b_n \left(- 3x_2 x_{n-1}^2 + k_{n-1,n} x_{n-1}^2 - 6(k_n - a_1) x_n x_n + \left(k_{n-1} - a_2\right) x_{n-1}\right)$$

$$\frac{\partial V_n(x)}{\partial x_n} = b_n \left(x_n^3 + k_{n,n-1} x_n^2 - 6(k_n - a_1) x_n x_{n-1} - \left(k_n - a_1\right) x_n\right)$$

Let's they represent Lyapunov's function in a scalar form in the following view:

$$\begin{aligned} V(x) = -b_n x_2 x_1^3 + \frac{1}{3} b_n k_{12} x_1^3 - 3b_n (k_1 - a_n) x_1^2 x_2 + b_n \frac{1}{2}\left(k_1 - a_n\right) x_1^2 \\ + \frac{1}{4} b_n x_2^4 + \frac{1}{3} b_n k_{12} x_2^3 - 3b_n \left(k_1 - a_n\right) x_1 x_2^2 + \frac{1}{2} b_n (k_2 - a_{n-1}) x_2^2 +, \dots, + \\ - b_n x_n x_{n-1}^3 + b_n k_{n-1,n} x_{n-1}^3 - 3b_n \left(k_n - a_1\right) x_n x_{n-1}^2 + \frac{1}{2} b_n \left(k_{n-1} - a_2\right) x_{n-1}^2 + \\ + \frac{1}{4} b_n x_n^4 + \frac{1}{3} b_n k_{n,n-1} x_n^3 - 3b_n \left(k_n - a_1\right) x_{n-1} x_n^2 - \frac{1}{2} b_n (k_n - a_1) x_n^2 - \\ - \frac{1}{2} x_2^2 - \frac{1}{2} x_3^2 -, \dots \; , -\frac{1}{2} x_n^2 \end{aligned} \qquad (17)$$

Conditions of positive or negative definiteness of function (17) can't be defined therefore we will use a lemma of the Mors and locally we can present function (17) in locality of a steady state by the type of a square form:

$$\begin{aligned} V(x) \approx \frac{b_n}{2} \Bigg[\left(k_1 - a_n\right) x_1^2 + \left(k_2 - a_{n-1} - \frac{1}{b_n}\right) x_2^2 + \left(k_3 - a_{n-2} - \frac{1}{b_n}\right) x_3^2 + \\ + \left(k_4 - a_{n-3} - \frac{1}{b_n}\right) x_4^2 +, \dots, + \left(k_n - a_1 - \frac{1}{b_n}\right) \Bigg] \end{aligned} \qquad (18)$$

The necessary condition of stability of a steady state (8) will be defined by system of inequalities at $b_n > 0$:

$$\begin{cases} k_1 - a_n > 0 \\ k_2 - a_{n-1} - \dfrac{1}{b_n} > 0 \\ k_3 - a_{n-2} - \dfrac{1}{b_n} > 0 \\ k_4 - a_{n-3} - \dfrac{1}{b_n} > 0 \\ \dots \quad \dots \quad \dots \\ k_{n-1} - a_2 - \dfrac{1}{b_n} > 0 \\ k_n - a_1 - \dfrac{1}{b_n} > 0 \end{cases} \qquad (19)$$

From system of inequalities (15) and (19) it is obvious that the control system with an increased potential of robust stability provides stability to system (3) at any changes of uncertain parameters.

V. CONCLUSIONS

In this work universally a new approach to create of Lyapunov's vector functions is offered.

Components of a vector of an anti-gradient of vector functions from geometrical interpretation of the theorem of the second method of Lyapunov are set by components of a speed vector (the right member of a state equation).

Research of robust stability of a system is made by designing of some negative function equal to a scalar product of a vector of gradients on a vector of speed.

Stability conditions turn out from positive definiteness of Lyapunov's vector function, in the form of system of inequalities in uncertain parameters of objects of control and established parameters of the regulator.

Known methods of control systems creation with uncertain parameters are generally devoted to determination of robust stability of system with the set structure with linear laws of control or radiant nonlinear (relay) characteristics and don't allow to project a control system with rather wide area of robust stability in the conditions of big uncertainty of parameters of object of control and drift of their characteristic in big limits.

Actually the results received on creation of systems control with an increased potential of robust stability, allow to ensure dynamic safety and operability of operated systems at a stage of their designing and operation. Usage of the developed approach to create of Lyapunov's functions allowed to demonstrate that the system has an asymptotically steady stationary states both in negative, and in positive area of change of uncertain parameters of control object.

When transiting of uncertain parameters through zero there is a bifurcation and new steady branches appear. Thus the zero steady state loses stability.

These stationary states at the same time don't exist and there is an opportunity to synthesize steady system at any change of uncertain parameters.

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